Preprint YERPHI-1470(7)-96

HIGGS BOSON MASSES IN NMSSM WITH SPONTANEOUS CP-VIOLATION

ASATRIAN H.M., YEGHIYAN G. K.

Yerevan Physics Institute, Alikhanyan Br. 2, Yerevan, Armenia
e-mail: "asatryan@vx1.yerphi.am"

Abstract

The Higgs boson mass problem is considered in the next-to-minimal supersymmetric standard model for the case of the spontaneous CP violation. The renormalization group equations for the gauge, Yukawa and scalar coupling constants, the effective Higgs potential and lower experimental bounds on Higgs boson and chargino masses are analyzed. The restrictions on the Higgs boson masses are found.

1.The aim of our paper is to consider the problem of Higgs boson masses in the next to minimal supersymmetric standard model (NMSSM) for the case of the presence of spontaneous CP violation. Such a model contains an additional Higgs singlet, as compared with the minimal supersymmetric standard model (MSSM) [1]. It is known, that while in MSSM one can't realize the realistic scenario with spontaneous CP-violation [2], in NMSSM such a scenario, generally speaking, is realizable [3, 4]. However, experimental bounds on the Higgs masses strongly restrict the space of parameters of theory, where such a scenario could be realized. In this paper we will continue the consideration of the Higgs boson masses problem for the case with spontaneous CP violation in NMSSM, taking into account the existing

experimental restrictions on the Higgs boson and chargino masses and using the renormalization group equations and Higgs effective potential analysis. This allows us to obtain more restrictive bounds on Higgs boson masses than those in Ref. [3].

2.The Higgs sector of NMSSM consists of two Higgs doublets $H_1 = \begin{pmatrix} \xi_1^+ \\ \xi_1^o \end{pmatrix}$, $H_2 = \begin{pmatrix} \xi_2^o \\ \xi_2^- \end{pmatrix}$ with hypercharges $Y(H_1) = 1, Y(H_2) = -1$ and the complex $SU(2)_L \times U(1)_Y$ singlet N. The superpotential for one quark generation is the following [5]:

$$W = \frac{\lambda_1}{3} N^3 + \lambda_2 N H_1 H_2 + h_u H_2 Q_L^c u_R + h_d H_1 Q_L^c d_R \tag{1}$$

where

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad Q_L^c = i\sigma_2 Q_L^*$$

The scalar potential for the Higgs fields is the following [3, 5, 7, 8]

$$\begin{aligned} V &= \frac{1}{2} a_1 (H_1^+ H_1)^2 + \frac{1}{2} a_2 (H_2^+ H_2)^2 + a_3 |H_1|^2 |H_2|^2 + a_4 |H_1 H_2|^2 + a_5 |N|^2 |H_1|^2 \\ &+ a_6 |N|^2 |H_2|^2 + a_7 ((N^2)^+ H_1 H_2 + h.c.) + a_8 |N|^4 + + m_4 (N H_1 H_2 \\ &+ h.c.) + \frac{m_5}{3} (N^3 + h.c) + m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 |N|^2 \end{aligned} \tag{2}$$

The parameters of the potential (2) are connected by the relations (for the more detailed discussion of the results, represented in this chapter, see [3]):

$$\begin{aligned} a_5 &= a_6 = \lambda_2^2, \quad a_7 = \lambda_1 \lambda_2, \quad a_8 = \lambda_1^2 \\ a_1 &= a_2 = \frac{1}{4}(g_1^2 + g_2^2), \;\; a_3 = \frac{1}{4}(g_2^2 - g_1^2), \;\; a_4 = \lambda_2^2 - \frac{1}{2} g_2^2 \end{aligned} \tag{3}$$

where g_1, g_2 are gauge coupling constants of the gauge groups $U(1)_Y$ and $SU(2)_L$ respectively. These relations are valid for the supersymmetry breaking scale M_s and higher. Below the supersymmetry breaking scale the energy behavior of the coupling constants $a_i (i = 1, 2, ..., 8)$ (and also of the Yukawa and gauge couplings) is given by renormalization group equations. The mass parameters of the potential (2) m_j, j=1,..,5 are connected with the supersymmetry breaking. Unlike Refs [5, 6, 7, 8] we consider the model, where supersymmetry breaking terms are not universal [4, 9]. This means that the parameters m_j are independent.

If the Higgs fields H_1, H_2, N in potential (2) develop nonzero VEV's, the electroweak symmetry breaking takes place. To provide for the electric charge conservation we choose these VEV's in the following form:

$$< H_1 >= \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \qquad < H_2 >= \begin{pmatrix} v_2 e^{i\varphi} \\ 0 \end{pmatrix} \qquad < N >= v_3 e^{i\alpha}$$

The case of the spontaneous CP-violation corresponds to vacuum with nonzero phases: $\varphi \neq 0$ $\alpha \neq 0$.

After the spontaneous breaking of electroweak symmetry five neutral and one complex charged scalar fields appear. Excluding the Goldstone mode, we obtain 5×5 symmetric mass matrix for neutral fields Φ_1, Φ_2, A, N_1, N_2, where CP-even fields Φ_1, Φ_2 and CP-odd field A are contained by Higgs doublets H_1, H_2 and CP-even field N_1 and CP-odd field N_2 are contained by the singlet N. The mass of charged Higgs is given by:

$$m^2_{H^+} = -\frac{2(m_4 cos(\varphi + \alpha) + a_7 v_3 cos(\varphi - 2\alpha))v_3}{sin 2\beta} - a_4 \eta^2 \tag{4}$$

where

$$\eta^2 = v_1^2 + v_2^2 = (174 GeV)^2, \tag{5}$$

and

$$1 < \tan\beta = \frac{v_2}{v_1} = \frac{h_b}{h_t}\frac{m_t}{m_b} < 60 \tag{6}$$

where [10, 11] $m_t = (175 \pm 15) GeV$, $m_b = (3.5 \pm 0.5) GeV$ are t- and b-quark masses respectively and $1 \leq h_t \leq 1.1$, $h_b \leq 1.1$ are their Yukava coupling constants.

To obtain the restrictions on Higgs particles masses, we have investigated the restrictions on parameters of the Higgs potential (2). The restrictions for coupling constants $a_i, i = 1, ...8$ can be obtained, analyzing the renormalization group equations for $a_i, i = 1, ...8$, gauge $g_r, r = 1, 2, 3$ and t- and b-quark Yukawa couplings h_t, h_b (one can neglect other Yukawa couplings because of their smallness with regard to h_t and h_b) in the region between electroweak breaking scale and the supersymmetry breaking scale and from the supersymmetry breaking scale up to unification scales ($\sim (10^{16} - 10^{18}) GeV$). We assume that all of the (gauge, Yukawa, scalar) coupling constants of our theory are small between weak and unification scales, so that the perturbation theory applies [3, 12, 13]. Besides this, for the theory to be a correct one,

the condition of vacuum stability is necessary. These conditions give some restrictions for values of scalar coupling constants a_i at $Q = M_Z$. The Higgs VEV's v_1, v_2 are restricted by conditions (5),(6). The mass parameters m_j, j=1,...,5 are connected with the supersymmetry breaking and are of the order of supersymmetry breaking scale or smaller. The restrictions on parameters of theory are also obtained from minimum conditions on the Higgs potential (2) and from the requirement of positiveness of Higgs boson masses squared. Using above mentioned restrictions on parameters of theory, we obtain that in the case of the absence of spontaneous CP-violation the lightest neutral Higgs boson has the mass of order of η or smaller. The mass of this particle depends on the radiative corrections to the scalar coupling constants: due to these corrections one increases about 40-60 GeV , depending on supersymmetry breaking scale. The remaining Higgs particles, however, can be as heavy as the supersymmetry breaking scale. This situation is similar to one in the minimal supersymmetric standard model. In spite of the MSSM has smaller number of independent parameters, only for the lightest Higgs boson an upper bound on the mass can be obtained. It is obtained that this particle always lighter than Z-boson, if the tree-level potential is analyzed. However, due to the radiative corrections to the MSSM Higgs potential the mass of the lightest Higgs boson increases significantly (see for example [6] and references therein).

In the case of the presence of spontaneous CP-violation situation is much more interesting. The requirement of positiveness of Higgs particles masses squared can be satisfied, only if the supersymmetry breaking scale is higher than 100GeV. In other words, we come to conclusion that the scenario with spontaneous CP-violation can be realized, only if the supersymmetry breaking scale is higher than 100GeV. In this case at least three neutral detectable (nonsinglet) Higgs particles exist with masses of the order of η or smaller. The charged Higgs boson also has the mass of this order. As for the two remaining neutral Higgs bosons, masses of these ones can be as large as the supersymmetry breaking scale. However, two heaviest neutral Higgses must be almost $SU(2)_L \times U(1)_Y$ singlets, if they have masses much larger than η.

Such a difference between the cases of the presence and the absence of spontaneous CP-violation takes place due to the additional restriction on parameters of theory in the case of the presence of spontaneous CP-violation, which follows from the minimum conditions for nonzero vacuum phases . This restriction, combined with the requirement of positiveness of Higgs particles

masses squared, leads to the following restriction on parameters of theory

$$
\begin{aligned}
& max((a_3 + a_4 - \sqrt{a_1 a_2})\frac{\eta^2 \sin 2\beta}{2v_3^2}, \frac{a_7^2 \eta^2 \sin 2\beta}{2a_8 v_3^2}) < \\
& < -a_7 \frac{\sin 3\alpha}{\sin(\varphi + \alpha)} < (a_3 + a_4 + \sqrt{a_1 a_2})\frac{\eta^2 \sin 2\beta}{2v_3^2}
\end{aligned} \tag{7}
$$

which in his turn leads to results, described above

3. Besides this, we have found in [3] that in the case of the presence of spontaneous CP-violation the lightest neutral Higgs (detectable or no) has the mass

$$
m_h \le 35 GeV \tag{8}
$$

On the other hand the experimental restrictions on Higgs boson masses in the minimal supersymmetric standard model are the following [14, 15]:

$$
m_h > 45 GeV, \qquad\qquad m_{H^+} > 45 GeV \tag{9}
$$

for CP-even and charged Higgses respectively and

$$
\begin{aligned}
m_A &> 27 GeV \quad for \quad 1 < \tan\beta < 1.5, \\
m_A &> 45 GeV \quad for \quad \tan\beta > 1.5
\end{aligned} \tag{10}
$$

for the CP-odd Higgs A. Unlike the MSSM, NMSSM contains an additional complex $SU_L(2) \times U_Y(1)$ singlet field N so that in NMSSM with the presence of spontaneous CP-violation the lightest neutral Higgs, generally speaking, is some mixing of five real scalar fields: two CP-even and one CP-odd fields, contained by Higgs doublets H_1 and H_2, and two singlet fields N_1,N_2. This fact and conditions (8)-(10) lead us to the following conclusion.

The scenario with spontaneous CP-violation in NMSSM, generally speaking, can be realistic for $\tan\beta > 1$, if the lightest neutral Higgs boson is almost $SU_L(2) \times U_Y(1)$ singlet. Also the scenario with spontaneous CP-violation for $1 < \tan\beta < 1.5$ can be realistic, if the lightest neutral Higgs is some mixing of the CP-odd field A and singlet fields N_1 and N_2. However, the last scenario seems to be improbable. First of all, the new experiments soon can exclude also the possibility of existing of neutral Higgs with the mass smaller than 35GeV. Besides this, the region of $1 < \tan\beta < 1.5$ is permissible only for $m_t \approx 160 GeV$, i.e. for the minimal value of t-quark mass, allowed by

[10]. The new experimental data [16] give the average value of t-quark mass $m_t = (180 \pm 12) GeV$. For a latter values of t-quark mass the allowable region of $\tan\beta$ is $1.5 < \tan\beta < 60$. This means that the lightest neutral Higgs must be almost $SU(2)_L \times U(1)_Y$ singlet, to avoid the contradiction with experiment. As it was mentioned above, in this case three detectable (i.e. nonsinglet) neutral Higgs particles with masses of the order of η or smaller always exist.

4.Thus we must investigate the case, when (in the case of the presence of spontaneous CP-violation) the lightest neutral Higgs is almost singlet. The spectrum of the neutral Higgs masses is described by the 5×5 mass matrix M^2, given in [3] by formula (A1) in Appendix A. This matrix was obtained for the vector H, which has the form $H^T = (\Phi_1, \Phi_2, A, N_1, N_2)$. The lightest neutral Higgs boson is the some linear combination of fields Φ_1, Φ_2, A, N_1, N_2, i. e.

$$h = x_1\Phi_1 + x_2\Phi_2 + x_3 A + x_4 N_1 + x_5 N_2$$

so that the coefficients x_k, k=1,..,5 can be found from the equation

$$M^2 X_1 = m_h^2 X_1 \tag{11}$$

where $X_1^T = (x_1, x_2, x_3, x_4, x_5)$. The requirement that the lightest neutral Higgs boson must be almost $SU(2) \times U(1)$ singlet means that the following condition must be satisfied:

$$\frac{x_4^2 + x_5^2}{|X_1|^2} \to 1 \tag{12}$$

The analytical investigation of equation (11) shows that at least two cases exist, when the condition (12) can be satisfied. These cases are the following:

$$a) \qquad a_7 v_3 \ll \eta \quad and \quad v_3 \gg \eta \sin 2\beta \tag{13}$$

$$b) \qquad \varphi \ll \alpha \ll 1 \quad and \quad a_7 v_3^2 \sim \eta^2 \sin 2\beta \tag{14}$$

As one can observe, lower experimental bounds on the Higgs boson masses give additional restrictions on the singlet VEV v_3. On the other hand, it is known that the singlet Higgs boson interacts also with charged and neutral Higgs fermions (Higgsinos). This means that the restrictions (13) and (14) on v_3 can bring some new restrictions on chargino and neutralino masses. In particular, it can happen that in parameter space, satisfying the conditions

(13) and (14), charginos and neutralinos will have masses, which are smaller than existing lower experimental bounds on their values [17]. An analytical and numerical investigations show that while the lower experimental bounds on neutralino masses don't give some essential restrictions on the Higgs boson masses, the experimental bound on the lightest chargino mass $m_{c_1} > 45GeV$ plays crucial role. Therefore in the next chapter we will consider the chargino masses problem in more detail.

5.The method of finding chargino masses in the minimal supersymmetric standard model is described in Ref. [18]. The difference between our case and the MSSM is that we must make replacements $v_2 \to v_2 e^{i\varphi}$ and $\mu \to \lambda_2 v_3 e^{i\alpha}$, where $\lambda_2 \approx \lambda_2(M_s)$. Then the lightest chargino mass square is given by

$$\begin{aligned} m^2_{C_1} &= \frac{1}{2}(M_2^2 + \lambda_2^2 v_3^2 + g_2^2\eta^2 - ((M_2^2 + \lambda_2^2 v_3^2 + g_2^2\eta^2)^2 - 4M_2^2\lambda_2^2 v_3^2 - \\ &- g_2^4\eta^4 \sin^2 2\beta + 4M_2\lambda_2 v_3 g_2^2\eta^2 \sin 2\beta \cos(\varphi+\alpha))^{1/2}) \end{aligned} \tag{15}$$

where M_2 is SU(2) gaugino mass arising from supersymmetry breaking.

Figure 1 presents the minimal value of $\lambda_2 v_3$ as a function of $\tan\beta$ for which the condition $m_{C_1} > 45GeV$ [17] is satisfied. These restrictions on $\lambda_2 v_3$ was found from the numerical analysis of (15). As it follows from Fig. 1,

$$\lambda_2 v_3 > 45GeV \tag{16}$$

for $\tan\beta > 10$. Thus we have shown that the lower experimental bound on the lightest chargino mass gives new restriction on the singlet VEV v_3.

6.Let us now consider the cases a) and b) in more detail. Before doing it we want to stress the following. It can be obtained from the renormalization group equations analysis that $a_7 \approx \lambda_1(M_s)\lambda_2(M_s)$. Also analytical and numerical investigations of the mass matrix M^2 show that the requirement of positiveness of Higgs boson masses squared can't be satisfied, if $\lambda_1 \ll \lambda_2$ (numerically $\lambda_1 < 0.1\lambda_2$). Taking into account two above mentioned conditions, we proceed now to consideration of cases a) and b). We obtain that in the case a) the conditions

$$a_7 \ll 1, \quad \lambda_2 \ll 1 \tag{17}$$

must be satisfied. Really, as it follows from (13), $v_3 \gg \eta$ for small values of $\tan\beta$ so that the necessity of the condition (17) is obvious. As for the large

values of $\tan\beta$, then (17) is necessary to the restriction (16) to be satisfied. Consequently, we obtain in this case that two neutral Higgses are almost singlets: one, the CP-odd Higgs boson N_2 has the mass of the order of few GeV or smaller, and the mass of the CP-even Higgs N_1 can be as heavy as the supersymmetry breaking scale. One of the detectable (nonsinglet) neutral Higgses is almost CP-odd field A. Using the condition (7), we obtain that for the mass of A the following condition must be satisfied:

$$m_A^2 < (a_4 + a_3 + \sqrt{a_1 a_2})\eta^2 \tag{18}$$

If $\tan\beta < 20$, then from the results, obtained from the analysis of renormalization groups equations for scalar coupling constants a_1, a_2, a_3, a_4 for the case $\lambda_2 \ll 1$ [3] we obtain that $m_A < 46GeV$ and $m_A < 52GeV$ for $M_s \sim 1TeV$ and $M_s \sim 10TeV$ respectively. In other words, the scenario with spontaneous CP-violation, described above, is disfavored for $\tan\beta < 20$. For larger values of $\tan\beta$ the mass of A increases due to increasing of a_1. It is also necessary to stress that for large values of $\tan\beta$ masses of two other nonsinglet neutral Higgses are given by

$$m_{h_1} \approx m_A \quad m_{h_2} \approx 2a_2\eta^2 \tag{19}$$

and these particles are almost CP-even fields Φ_2 and Φ_1 respectively. Last result (19) can be obtained, putting in nonsinglet part of the mass matrix $M^2 \sin 2\beta \approx 0$ and $\cos 2\beta \approx 1$.

As for the chargino and neutralino masses, they can be as heavy as the supersymmetry breaking scale, if $\lambda_1(M_s)$, $\lambda_2(M_s)$, and consequently a_7, are sufficiently small.

Notice also that in the case a) the condition $\varphi \gg \alpha$ must be satisfied, if $\tan\beta \gg 1$ or if $\lambda_2 v_3 \gg \eta$.

In the case b) ($\varphi \ll \alpha \ll 1$) the conditions (14), (16) and the requirement of positiveness of Higgs boson masses squared can't be satisfied simultaneously. The case b) is valid only for small values of $\tan\beta$, where the condition (16) is not valid. In this case the mass of CP-odd Higgs particle A is further restricted by the condition (18). However, as the numerical analysis of this case shows, this particle, generally speaking, can be heavier than 50GeV. As in the case a) the lightest singlet Higgs is almost CP-odd field N_2. Three remaining neutral Higgses are almost some mixings of CP-even fields Φ_1, Φ_2 and N_1.

7.As it could be seen from the previous analysis, in this model the nonsinglet Higgs boson masses strongly depend on the radiative corrections to the scalar coupling constants. The several approaches exist to take into account radiative corrections to the Higgs boson masses. One of them is the renormalization group equations analysis approach, which was used in ref. [3] and here up to this chapter. The other one is an effective potential approach, where except of corrections to the coupling constants the new terms, arising from the radiative corrections to the Higgs potential, are also taken into account [19, 20, 21]. In spite of the last approach more perfectly takes into account the radiative corrections, in NMSSM without spontaneous CP-violation both of approaches give almost the same results (compare, for example, the results of [3, 5] and [6]). It is interesting to investigate what does take place in the case of the presence of spontaneous CP-violation. In this chapter we will consider an effective potential approach, taking into account only t- and b-quark and squark one-loop corrections because of the largeness of h_t and (for large $\tan\beta$) h_b compared with other couplings constants. This means that we take into account only the loops, containing the following vertexes:

$$\begin{aligned}V_{tb} = h_t^2(|\tilde{Q}_L^+ H_2|^2 + |H_2|^2|\tilde{t}_R|^2) + h_b^2((|\tilde{Q}_L^+ H_1|^2 + |H_1|^2|\tilde{b}_R|^2) + \\ + h_t h_b(\tilde{t}_R^+ H_2^+ H_1 \tilde{b}_R + h.c) + (h_t \bar{Q}_L H_2 t_R + h_b \bar{Q}_L H_1 b_R + h.c)\end{aligned}$$

However, we continue take into account the radiative corrections, connected also with other coupling constants, using the renormalization groups analysis approach, where we rewrite the renormalization groups equations for the scalar coupling constants a_i, i=1,...,8 (see the preprint version of [3]) without terms, containing only h_t and h_b. To calculate the scalar loops contributions we use the method, described in Ref. [19]. Proceeding from supersymmetry, the contribution of quark loops is the same as the contribution of respective squark loops, taken with $m^2 = 0$, where m^2 is supersymmetry breaking squark masses (for a simplicity we take them equal). The method of finding the contribution of these loops to the masses of Higgs bosons is also well-known (see, for example, [20] and references therein). Using these methods, we must do the following. The restrictions on coupling constants must be found in the way described in the chapter 2. Then the transformations

$$a_1 \quad \rightarrow \quad a_1 + \frac{3}{8\pi^2} h_b^4 \ \ln(1 + \frac{m^2}{m_b^2})$$

$$
\begin{aligned}
a_2 &\rightarrow a_2 + \frac{3}{8\pi^2} h_t^4 \ \ln(1 + \frac{m^2}{m_t^2}) \\
a_3 &\rightarrow a_3 + \frac{3}{8\pi^2} h_b^2 h_t^2 \ \ln(1 + \frac{m^2}{m_t^2}) \\
a_4 &\rightarrow a_4 - \frac{3}{8\pi^2} h_b^2 h_t^2 \ \ln(1 + \frac{m^2}{m_t^2})
\end{aligned}
\tag{20}
$$

in Higgs boson mass-matrices must be done. It is clear that after this procedure the qualitative analysis of the restrictions on the Higgs boson masses, done in ref. [3], generally speaking, remain true. The difference is only that due to the transformations (20) the requirement of positiveness of the Higgs boson masses squared can be satisfied now also for $M_s \sim 100GeV$, so that the scenario with spontaneous CP-violation can be realized also at this scale. For $M_s \sim 1TeV$ and $M_s \sim 10TeV$ the following quantitative changes of the restrictions on the Higgs boson masses take place. In spite of $m_t^2 \sim M_Z^2$, due to the fact that $h_t(Q^2) < h_t(M_Z)^2)$, if $Q^2 > M_Z^2$ (this inequality is obtained from the renormalization group analysis for h_t) and, consequently,

$$
\int_0^{\frac{m^2}{M_Z^2}} \quad h_t^4(Q^2) d(\ln \frac{Q^2}{M_Z)^2}) < h_t^4(M_Z)^2) \ln \frac{m^2}{M_Z^2} \tag{21}
$$

Higgs boson masses can be larger than it is obtained, when the renormalization group equations analysis approach is used (compare the transformations (20) and the renormalization group equations for a_i, i=1,...4). It is clear that such a difference is most significant for $M_s \sim 10TeV$ (see formulae (25)-(27) in the next chapter). Such a difference can be obtained also in the case of the absence of spontaneous CP-violation, if the supersymmetry breaking scale $M_s \sim 10TeV$ is considered. However, for $M_s \sim 1TeV$ this difference is invisible [3, 5, 6]. The same result for $M_s \sim 1TeV$ is obtained here in the case of the presence of spontaneous CP-violation for small values of $\tan\beta$ (see formula (25) in the next chapter). For large values of $\tan\beta$ an additional increasing of masses of neutral Higgs bosons A, h_1 and charged Higgs boson H^+ takes place because now due to the largeness of h_b b-quark loops contribution to above mentioned particles masses is also significant (it is seen from (19) and (20) the mass of the Higgs boson h_2 doesn't depend on the b-quark and squark one-loop corrections). The same inequality as (21) can be written also for b-quark coupling constant h_b. However, it is necessary to

stress that such an inequality for h_b is much stronger because of $m_b \ll M_Z$ so that for large values of $\tan\beta$ the above mentioned increasing of Higgs boson masses is visible also for $M_s \sim 1TeV$ (see formulae (28)-(29) in the next chapter). Summarizing the above discussion, we come to conclusion that in the case of the presence of spontaneous CP-violation the restrictions on Higgs boson masses is less severe, if an effective potential approach applies.

8.Let us now consider the numerical results, which are obtained, if an effective potential approach, as more general method, applies. As we had mentioned above, in this case the lightest Higgs boson must be almost $SU(2)_L \times U(1)_Y$ singlet to avoid the contradiction with experiment (as the numerical investigations show the restriction on the lightest neutral Higgs $m_h < 35GeV$ remain valid, if an effective potential method applies). During the numerical investigations two cases were considered: first, when the lightest neutral Higgs boson is at least 90% singlet (case I) and the second, when it is at least 99% singlet (case II). For the case I we obtain that the lightest Higgs boson has mass $m_{h_s} < 0.5GeV, 1GeV, 9GeV$ for $M_s \sim 100GeV, 1TeV, 10TeV$ respectively. For the case II we find that $m_{h_s} < 0.2GeV, 0.8GeV, 3GeV$ for $M_s \sim 100GeV, 1TeV, 10TeV$ respectively. The existence of such a neutral Higgs bosons with such masses is not excluded by experiment [16].

The upper bounds as functions of $\tan\beta$, obtained for masses of two lightest nonsinglet mixings of CP-even fields Φ_1, Φ_2, N_1 (h_1 and h_2 respectively), for the mass of neutral CP-odd Higgs field A and for the mass of charged Higgs H^+ are presented in Fig. 2,3: in the Fig. 2 for the case I and in the Fig. 3 for the case II. As it was expected, the scenario with spontaneous CP-violation is possible now also for the supersymmetry breaking scale $M_s \sim 100GeV$. However, while for $M_s \sim 10TeV$ all of the values of $1.5 < \tan\beta < 60$ are allowable, for $M_s \sim 1TeV$ and $M_s \sim 100GeV$ the region of values of $\tan\beta$ exists, where the scenario with spontaneous CP-violation can't be realized. Besides this, although both in the case I and in the case II general restrictions on the detectable (nonsinglet) Higgs particles masses are almost the same (with accuracy of 10%), the restrictions for separate values of $\tan\beta$ in the case II are much stronger. In particular, for $M_s \sim 1TeV$ the region of excluded values of $\tan\beta$ in the case II is larger. The main cause of such a difference is the existence in the case I of additional allowable area of parameter space of theory: $\varphi, \alpha \ll 1$, $M_{35}^2 < \sqrt{10}M_{33}^2$, which was found

numerically. As it follows from Fig.2,3

$$
\begin{aligned}
m_{h_1} < 80GeV, &\quad m_A < 110GeV, \\
m_{h_2} < 105GeV, &\quad m_{H^+} < 110GeV
\end{aligned}
\tag{22}
$$

for $M_s \sim 100GeV$,

$$
\begin{aligned}
m_{h_1} < 90GeV, &\quad m_A < 115GeV, \\
m_{h_2} < 140GeV, &\quad m_{H^+} < 140GeV
\end{aligned}
\tag{23}
$$

for $M_s \sim 1TeV$,

$$
\begin{aligned}
m_{h_1} < 135GeV, &\quad m_A < 140GeV, \\
m_{h_2} < 180GeV, &\quad m_{H^+} < 185GeV
\end{aligned}
\tag{24}
$$

for $M_s \sim 10TeV$. It is interesting to compare the numerical results, obtained using the renormalization group equations analysis approach (method 1) with the results, obtained above (method 2). We do this for two cases: for low values of $\tan\beta$ ($\tan\beta < 10$) and for large values of $\tan\beta$ ($\tan\beta > 20$). For low values of $\tan\beta$

$$
\begin{aligned}
m_{h_1} < 90GeV, &\quad m_A < 115GeV, \\
m_{h_2} < 140GeV, &\quad m_{H^+} < 90GeV
\end{aligned}
\tag{25}
$$

for $M_s \sim 1TeV$, both of approaches

$$
\begin{aligned}
m_{h_1} < 95GeV, &\quad m_A < 125GeV, \\
m_{h_2} < 155GeV, &\quad m_{H^+} < 90GeV
\end{aligned}
\tag{26}
$$

for $M_s \sim 10TeV$, method 1,

$$
\begin{aligned}
m_{h_1} < 105GeV, &\quad m_A < 130GeV, \\
m_{h_2} < 175GeV, &\quad m_{H^+} < 95GeV
\end{aligned}
\tag{27}
$$

for $M_s \sim 10TeV$, method 2.

For large values of $\tan\beta$

$$
\begin{aligned}
m_{h_1} < 60GeV, &\quad m_A < 65GeV, \\
m_{h_2} < 135GeV, &\quad m_{H^+} < 115GeV
\end{aligned}
\tag{28}
$$

for $M_s \sim 1TeV$, method 1

$$\begin{aligned} m_{h_1} < 85GeV, \quad m_A < 90GeV, \\ m_{h_2} < 135GeV, \quad m_{H^+} < 140GeV \end{aligned} \tag{29}$$

for $M_s \sim 1TeV$, method 2

$$\begin{aligned} m_{h_1} < 90GeV, \quad m_A < 95GeV, \\ m_{h_2} < 155GeV, \quad m_{H^+} < 145GeV \end{aligned} \tag{30}$$

for $M_s \sim 10TeV$, method 1,

$$\begin{aligned} m_{h_1} < 135GeV, \quad m_A < 140GeV, \\ m_{h_2} < 180GeV, \quad m_{H^+} < 185GeV \end{aligned} \tag{31}$$

for $M_s \sim 10TeV$, method 2.

It is seen that the results, represented by (25)-(31), are in agreement with the predictions, done in previous chapter.

Summarizing the above discussion, we must also note that the restrictions on Higgs boson masses, obtained here, are much stronger, compared with ones, obtained in [3].

9.Thus we have shown that the scenario with spontaneous CP- violation in NMSSM to be real, the lightest neutral Higgs have to be almost $SU(2)\times U(1)$ singlet. This particle has mass of the order of a few GeV or smaller. This possibility is not excluded by experiment. The charged Higgs boson has mass of the order of η or smaller and at least three detectable (i.e. nonsinglet) neutral Higgs bosons exist with masses of the same order.

So our analysis of the problem of Higgs masses in NMSSM with the spontaneous CP-breaking shows that the considering model leads to the predictions for Higgs particles masses, which can be verified experimentally in the near future.

The research described in this publication was made possible in part by Grant No MVU000 from the International Science Foundation.

One of authors (Asatrian H. M.) wants to thank ICTP High Energy Section, where this work was completed, for hospitality.

References

[1] H. P. Nilles, M. Srednicki, D. Willer Phys. Lett. 120B, 366 (1983).

[2] A. Pomarol, Phys. Lett B287, 331 (1992).

[3] H. M. Asatrian, G. K. Eguian, Mod. Phys. Lett. A10, 2943 (1995),
H. M. Asatrian, G. K. Yeghiyan, Preprint YERPHI-1448(18)-95, (hep-ph/9508263)

[4] A. Pomarol, Phys. Rev., 47D, 273 (1993).

[5] T. Elliot, S. King, P. White, Phys. Lett. 305B, 71 (1993).

[6] Y. Okado, Preprint KEK-TH-382 (1993).

[7] U. Ellwanger et al. Preprint LPTHE Orsay 95-04 (hep-ph 9502206).

[8] J. Ellis et al. Phys. Rev. D39, 844 (1989).

[9] N. Nilles, Phys. Rep. 110, 1 (1984).

[10] H.B.Jensen Proceedings of the XXVII Intern. Conf. on HEP (Glasgow, 1994), p. 3.

[11] M. Bando et al. Mod. Phys. Lett. A7, 3379 (1992).

[12] N. Cabibo et al, Nucl. Phys. B158, 295 (1978)

[13] H. Asatrian, A. Ioannisyan, S. Matinyan, Z. Phys. C61 265(1994)

[14] M. Pohl, Proceedings of the XXVII Intern. Conf. on HEP (Glasgow, 1994), p. 107.

[15] DELPHI Collaboration, CERN PPE/94-218

[16] 1995 Partial Update to the Review of Particle Properties

[17] T. Mashimo, Proceedings of the XXVII Intern. Conf. on HEP (Glasgow, 1994), p. 775.

[18] H. Haber and G. Kane, Phys. Reports 117, 77 (1985)

[19] M. B. Voloshin, K. A. Ter-Martirosian, The Theory of Gauge Interactions of Elementary Particles, Moscow (1984)

[20] J.A.Casas et al. CERN-TH 7334/94, IEM-FT-87/84, hep-ph/9407389

[21] J. Ellis, G. Ridolfi, F. Zwirner, Phys. Lett. B 262 (1991), 477

Fig. 1. Minimal value of $\lambda_2 v_3$ satisfying the condition $m_{c_1} > 45$ GeV

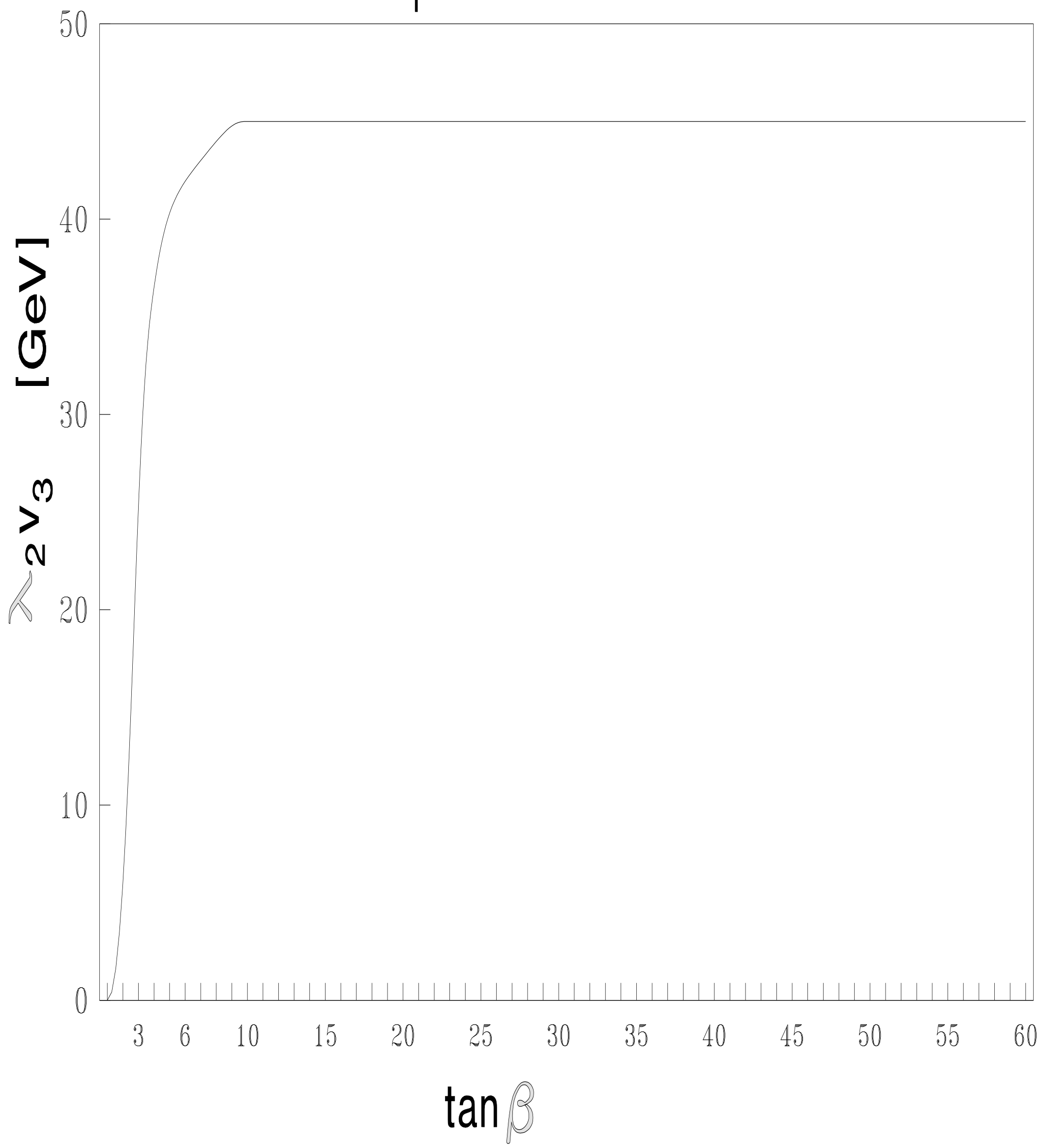

Fig. 2 Bounds on masses of nonsinglet Higgs bosons : case I

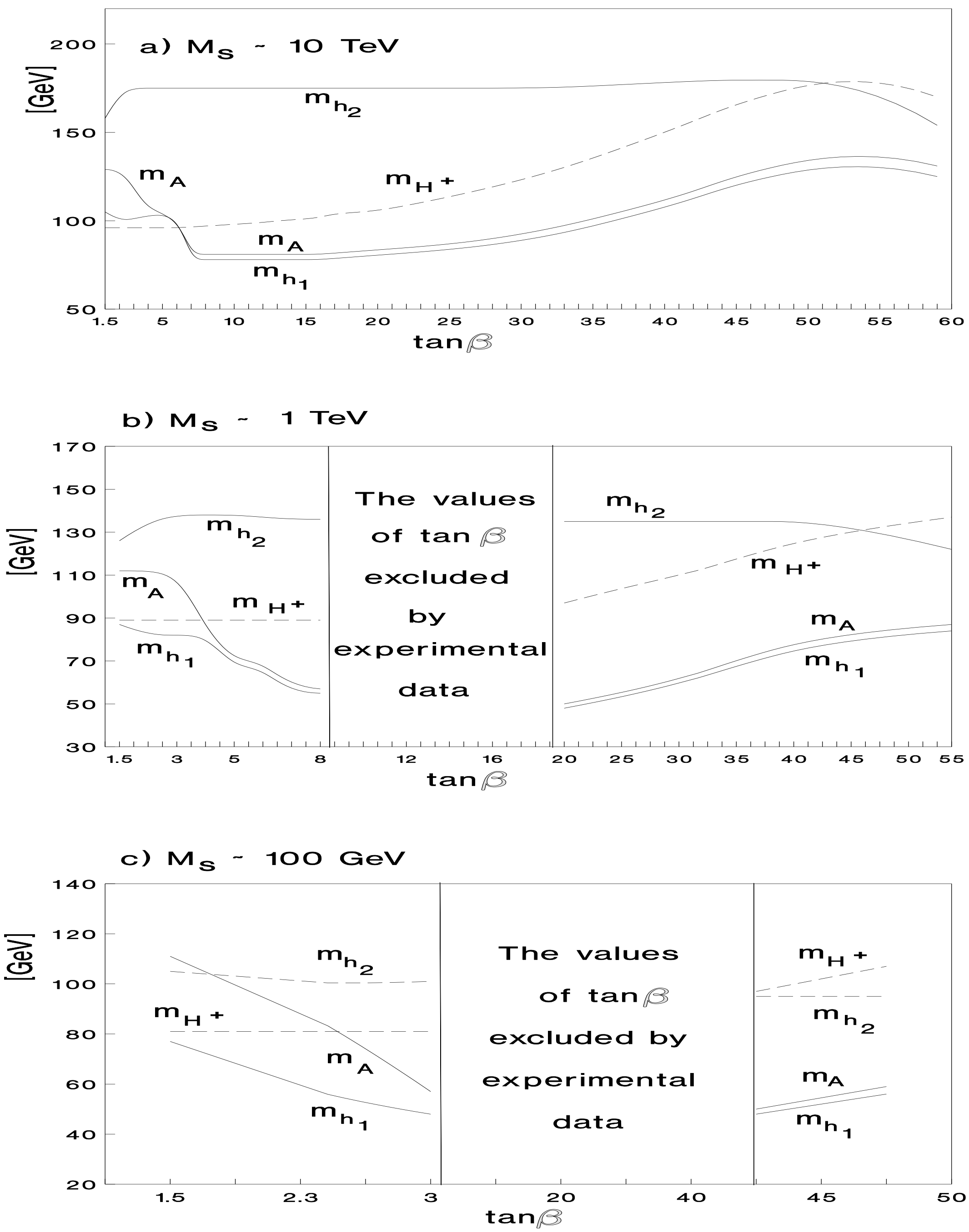

Fig. 3 Bounds on masses of nonsinglet Higgs bosons : case II